

Table 1. Developing innate thinking powers with questions

		<i>Innate thinking powers</i>	<i>Questions</i>	
Problem solving skills	Creating	Imagining the ability to visualise and to think about a current unknown in the sense of “What will happen if...?”	What do I notice? What do I wonder? What might I try to find out? How can I see this? What if...? I wonder what would happen if...?	Manipulating – getting a sense of – articulating to develop an increasingly Fluent understanding
		Expressing communicating mathematically by any means – talking, sketches, diagrams, charts, manipulating objects	What do I know? How can I show this? How else could I show this? What am I given? What do I need to find out?	
		Specialising taking what we know (or think we know) and trying it out (testing) (e.g. by creating specific examples) to gain information	What if...? I wonder what would happen if...? What am I given? What do I know? What examples can I give? Another? And another? Would this be an example? What is a non-example? What’s the same? What’s different?	
	Reasoning	Generalising (including: <i>analogic reasoning, relational reasoning</i> ³) identifying rules (generalisations) that can be applied to a wider range of situations, allowing us to make predictions and to use and apply knowledge to new problems	What’s the same? What’s different? What stays the same? What changes? What patterns can I see? Have I seen something like this before? What is this? Can I describe it in general terms? Is it always true? Can I show when this is true/not true?	
		Convincing first oneself and then others by using mathematical arguments and justifications, including providing specific examples to prove conjectures	How do I know? How can I show this? Is it always true? Why is this true? What is really going on here?	
		Conjecturing (<i>abductive reasoning</i> ³) suggesting relationships and making assertions based on the evidence gathered so far	What could be going on here? Why? What will happen if I...? Is there a relationship? Will it...?	
	Analysing	Classifying the process of identifying common properties or differences by which we can sort (e.g. data, objects, ideas)	What’s the same? What’s different? Have I seen something like this before? What is this? How can I define it?	
		Organising using a system or working systematically, (such as having a clear starting and finishing point to ensure all possibilities are examined)	What am I given? What do I know? What do I need to find out? What is a good way to start? What is the most helpful way to organise this?	

Table 2. Assessing innate thinking powers with example actions

		<i>Innate thinking powers</i>	<i>Example actions</i> > denote increasing sophistication but not a linear progression	
Problem solving skills	Creating	Imagining the ability to visualise and to think about a current unknown in the sense of “What will happen if...?”	Wondering > generating possibilities > seeking pattern > (mentally) trialling imposing structure > developing a narrative > visualising > visually restructuring/reorganising	Manipulating – getting a sense of – articulating to develop an increasingly Fluent understanding
		Expressing communicating mathematically by any means – talking, sketches, diagrams, charts, manipulating objects	Doing > manipulating > gesturing > showing > describing > representing > recording > emphasising > demonstrating > explaining > convincing > justifying > proving	
		Specialising taking what we know (or think we know) and trying it out (testing) (e.g. by creating specific examples) to gain information	Having a go > trying it out > purposeful trialling > experimenting > tinkering deliberately > changing as little as possible > accurately defining the example space > applying own constraints	
	Reasoning	Generalising (including: <i>analogic reasoning, relational reasoning</i> ³) identifying rules (generalisations) that can be applied to a wider range of situations, allowing us to make predictions and to use and apply knowledge to new problems	Repeating and forming expectations > getting a sense of > articulating (explaining in terms of what is already known) > applying (specialising) > proving (deductive reasoning³) defining – connecting representations – noticing variance & invariance – noticing structure – re-presenting – refining – redefining – broadening – narrowing – extending	
		Convincing first oneself and then others by using mathematical arguments and justifications, including providing specific examples to prove conjectures	Showing > describing > explaining > convincing > justifying (inductive reasoning³) > proving (deductive reasoning³)	
		Conjecturing (<i>abductive reasoning</i> ³) suggesting relationships (patterns) and making assertions based on the evidence gathered so far	Making and expressing connections > purposeful trialling (evidence of expectations) > interrogating relationships > predicting > generating and articulating hypotheses (“I think...” / “It’s going to...”)	
	Analysing	Classifying the process of identifying common properties or differences by which we can sort (e.g. data, objects, ideas)	Noticing features/properties > comparing > same > same/different > describing what is seen > naming > comparing > noticing variance/invariance > categorising > defining	
		Organising using a system or working systematically, (such as having a clear starting and finishing point to ensure all possibilities are examined)	Doing > ordering > mark making > deliberate recording > organising thoughts > planning (first, then) > organising information efficiently > being systematic taking care – checking - verifying	