Developing and assessing problem-solving in Year 6

As the main goal of maths education and one of the three aims of the national curriculum, it is important that all learners have the opportunity to develop their problem-solving throughout primary school. This guidance accompanies three suggested rich task activities that provide opportunities to develop and assess problem-solving using age-appropriate content from the national curriculum. **To get the most from these activities, teachers should first engage in the tasks themselves and, preferably, discuss with colleagues any potential responses, challenges, scaffolds and extensions for learners.** This will facilitate learners to get the most from the activities.

Observing and questioning learners as they engage in such activities, supports assessment of their problem-solving skills and their mastery of the content of the curriculum. These assessments can inform judgements against age-related expectations but can also identify next steps and provide opportunities for learners to share their strategies and reasoning, thus supporting learning.

Working at age-related expectations in Year 6 involves finding solutions to novel (i.e. unseen) problems that require thought and effort and the ability to *use and apply the range of mathematical content in the primary curriculum*. Successful problem-solvers make use of a number of strategies that can help them get started or break into a problem, work towards a solution and find ways of getting 'unstuck'. They may display perseverance and determination not to give up (learning dispositions) but it is making use of well-developed problem-solving skills that underpins success.

What problem-solving skills can be developed and assessed?

There are a number of different classifications of problem-solving skills but there are two key sets of primary-focused resources that are closely aligned (with some differences in language only):

- Due to the previous work on developing habits of mind, CLF resources use John Mason's ideas on "learners' innate thinking powers" as the basis for developing and assessing problem-solving (see tables 1 & 2 below).
- This article on the NRICH website also gives clear guidance on the problem-solving process and identifies a number of key skills that can be developed and assessed in learners.

Whichever model is used, learners should be aware of the kinds of strategies they are trying to develop and be given frequent opportunities to self-assess and to develop metacognitive skills to choose the most appropriate strategies for given problems. Rather than having the perfect ready-made formula for each kind of problem, learners should have a toolbox of questions, strategies and actions to choose from, meaning they are equipped for tackling all sorts of unfamiliar problems; what being a mathematician is all about.

Teachers can promote useful ways of thinking by modelling strategies and actions and by asking questions that, over time, learners can adopt as their own. The premise of "innate thinking powers" is that all learners have a natural aptitude for these ways of thinking (see the coloured pairs in Tables 1 & 2) so the role of the teacher is to draw attention to and encourage learners to make use of them. Collaborative investigative activities, rich tasks and novel problems provide opportunities for learners to develop these skills independently and with each other and should be a regular part of maths learning so that skills build over time. The NRICH website has many examples with supporting guidance and the ATM publications below contain more examples with resources and advice.

ATM publications: "Assessing in the new curriculum – An ATM perspective" "Thinking mathematically Every Day: Year 6" and many more.

Table 1. Developing useful habits of mind (strategies and questions) for solving problems

		Strategies (including 'innate thinking powers')	***Questions***
Problem solving skills	Creating	**Imagining** the ability to visualise and to think about a current unknown in the sense of "What will happen if…?"	What do I notice? What do I wonder? What might I try to find out? How can I see this? What if…? I wonder what would happen if…?
		Expressing communicating mathematically by any means – talking, sketches, diagrams, charts, manipulating objects	What do I know? How can I show this? How else could I show this? What am I given? What do I need to find out?
		Specialising taking what we know (or think we know) and trying it out (testing) (e.g. by creating specific examples) to gain information	What if…? I wonder what would happen if…? What am I given? What do I know? What examples can I give? Another? And another? Would this be an example? What is a non-example? What's the same? What's different?
	Reasoning	**Generalising** *(including: analogic reasoning, relational reasoning)* identifying rules (generalisations) that can be applied to a wider range of situations, allowing us to make predictions and to use and apply knowledge to new problems	What's the same? What's different? What stays the same? What changes? What patterns can I see? Have I seen something like this before? What is this? How can I describe it in general terms? Can I show when this is true/not true?
		Convincing first oneself and then others by using mathematical arguments and justifications, including providing specific examples to prove conjectures	How do I know? How can I show this? Is it always true? Why is this true? What is really going on here?
		Conjecturing *(abductive reasoning[4])* suggesting relationships and making assertions based on the evidence gathered so far	What could be going on here? Why? What will happen if I…? Is there a relationship? Will it…?
	Analysing	**Classifying** the process of identifying common properties or differences by which we can sort (e.g. data, objects, ideas)	What's the same? What's different? Have I seen something like this before? What is this? How can I define it?
		Organising using a system or working systematically, (such as having a clear starting and finishing point to ensure all possibilities are examined)	What am I given? What do I know? What do I need to find out? What is a good way to start? What is the most helpful way to organise this?
	Evaluating	**Reflecting[2]** *(metacognition)* evaluating ones thought processes and actions to identify next steps or future strategies	How am I approaching this? What else could I try? Have I seen something like this before? Is there a better way? Could I have done this differently? Was this the best way to start? What did I learn? How is this helpful to me?
		Extending[2] deliberately going beyond the original problem to learn and understand more	What if…? I wonder what would happen if…? What related questions could I explore?

Table 2. Developing useful habits of mind (strategies and actions) for solving problems

		Strategies (including 'innate thinking powers')	***Example actions*** *> denote increasing sophistication but not a linear progression*
Problem solving skills	**Creating**	**Imagining** the ability to visualise and to think about a current unknown in the sense of "What will happen if…?"	Wondering > generating possibilities > seeking pattern > (mentally) trialling imposing structure > developing a narrative > visualising > visually restructuring/reorganising
		Expressing communicating mathematically by any means – talking, sketches, diagrams, charts, manipulating objects	Doing > manipulating > gesturing > showing > describing > representing > recording > emphasising > demonstrating > **explaining** > **convincing > justifying > proving**
		Specialising taking what we know (or think we know) and trying it out (testing) (e.g. by creating specific examples) to gain information	Having a go > trying it out > purposeful trialling > experimenting > tinkering deliberately > changing as little as possible > **accurately defining the example space > applying own constraints**
	Reasoning	**Generalising** *(including: analogic reasoning, relational reasoning)* identifying rules (generalisations) that can be applied to a wider range of situations, allowing us to make predictions and to use and apply knowledge to new problems	Repeating and forming expectations > getting a sense of > articulating (explaining in terms of what is already known) > applying (specialising) > **proving** (*deductive reasoning*[4]) defining – connecting representations – noticing variance & invariance – noticing structure – re-presenting – refining – redefining – broadening – narrowing – extending
		Convincing first oneself and then others by using mathematical arguments and justifications, including providing specific examples to prove conjectures	Showing > describing > **explaining > convincing > justifying (***inductive reasoning*[4]***) > proving (***deductive reasoning*[4]**)**
		Conjecturing *(abductive reasoning*[4]*)* suggesting relationships and making assertions based on the evidence gathered so far	Making and expressing connections > purposeful trialling (evidence of expectations) > interrogating relationships > predicting > **generating and articulating hypotheses** ("I think…" / "It's going to…")
	Analysing	**Classifying** the process of identifying common properties or differences by which we can sort (e.g. data, objects, ideas)	Noticing features/properties > comparing > same > same/different > describing what is seen > naming > comparing > noticing variance/invariance > categorising > **defining**
		Organising using a system or working systematically, (such as having a clear starting and finishing point to ensure all possibilities are examined)	Doing > ordering > mark making > deliberate recording > organising thoughts > planning (first, then) > organising information efficiently > **being systematic** taking care – checking – verifying
	Evaluating	**Reflecting**[2] *(metacognition)* evaluating ones thought processes and actions to identify next steps or future strategies	Reviewing > developing awareness > noticing > monitoring > evaluating > connecting to previous experiences parking (the first idea) – experimenting – refining – practising
		Extending[2] deliberately going beyond the original problem to learn and understand more	Identifying similar problems – identifying related areas for exploration – constraining original task further – broadening the original task > further generalising

For additional guidance on developing and assessing habits of mind for problem solving (including learning dispositions and useful attitudes), please see the source document for these tables here.