Assessing learners as mathematicians

A common interpretation of the three aims of the national curriculum is that, first, we need to develop fluency, then encourage learners to reason about the concept (often focusing on the use of correct mathematical vocabulary based on highly-structured sentence frames) and that problem solving is something that comes later and can be seen as the 'icing on the cake'.

This interpretation sometimes results in the structuring of learners' daily independent work as follows:

PART A – Fluency: multiple similar, basic questions that repeat a procedure/skill or rehearse some basic facts;

PART B – Reasoning: instructions to write or say an explanation about how something is known or why something is correct or incorrect;

PART C – Problem solving: word problems (often directly linked to the LO or WALT) and/or sometimes a challenge/investigation/puzzle.

Over-emphasising the importance of recorded work and structuring the tasks in the above way can create problems. Those learners who haven't developed a good understanding of concepts will often start on the fluency exercise and never move any further. Additionally, if the independent work is the only point in the lesson that promotes reasoning and problem-solving, these children don't always get the opportunity to be mathematical.

How do we begin to assess learners holistically as mathematicians if they don't get opportunities to demonstrate (and develop) their reasoning and problem-solving?

Even those learners who do regularly access the reasoning and problem-solving tasks may be getting far less from them than if they were engaging in a whole class discussion about the different ways they have explained, understood or found solutions.

The premise that fluency needs to come first is the one that I wish to challenge. It's built on a skewed view of what fluency is and it leads to procedures and facts being the focus of much whole-class teaching and recorded work.

What is fluency?

To define fluency, first it is necessary to define understanding. Meaningful understanding of a concept is based on making generalisations about it; being able to define what it is and what it isn't so that we can "describe the concept in terms of other concepts that we already understand, in a way that allows [us] to use it"[1]. We come to these generalisations by reasoning about the ideas involved and discerning what is critical.

These generalisations are what allow us to move fluently between different representations and to make connections between related concepts – to recognise, for instance, that a problem situation has a mathematical structure that we can model with a multiplication calculation such as 9 × 7. Without generalising, it is not possible to have a fluent understanding of a concept. To summarise this more briefly, *reasoning underpins fluency*.

This does not mean that certain rote-learning style activities cannot be *involved in* building fluent understanding. For example, being able to chant the multiples of the three-times table, be familiar with the pattern of multiples of three and recognise that a given number is (or is not) a multiple of three are all important knowledge when developing understanding of multiplying and dividing by three. However, without making the necessary generalisations, without noticing the patterns and links, without making

[1] Efrat Furst (2019) "Meaning First" https://sites.google.com/view/efratfurst/meaning-first

connections to other concepts (e.g. seeing that non-multiples are multiple of three +/- 1 or 2), the ability to quickly use and apply that information is limited. Not non-existent; just limited. For example, given an exercise titled "Dividing by three", rote learners might be able to make use of their three times-table facts to correctly solve some calculations. However, would the same proportion of learners be able to recognise an application of multiplying by three to find the 20^{th} or n^{th} terms in the following number sequence? 8, 11, 14, 17,…

So how do we promote reasoning?

The simplest way we can promote reasoning is to *give learners more opportunities to grapple with and make sense of concepts for themselves.* We need to ask them questions that force them to think. And the greatest resource available to develop their reasoning and encourage them to refine their ideas is each other. In order to make sure learners are doing their own thinking and developing their understanding, we need to stop them relying on us to do it for them.

Most of the time we can't just tell learners what they need to know, since our explanation or definition is unlikely to match their previous conceptions closely enough to require no thought to be able to generalise about it. We can improve the ease with which learners can connect concepts and incorporate new information by using language and models that they are already familiar with and by giving them activities and tasks that actively encourage them to notice and think about the concept and how they understand it.

Developing learner-led (teacher-facilitated) whole-class dialogue using tasks and activities that prompt reasoning is the most efficient way of supporting all learners to develop a deeper understanding of concepts. If we avoid 'telling' learners as much as possible and always reflect their questions back to them (using discussion with other learners as a resource); if we avoid sanctioning individual responses as correct or incorrect; if we encourage all learners to express their understandings and examine differences to come to a shared understanding; if we manage to develop some or all of these ideas, we will be giving more learners the opportunity to reason.

And as one of the three aims of the national curriculum, it is something <u>all</u> learners are entitled to.

And what about problem solving?

True problem solving (finding solutions to previously *unseen tasks with unfamiliar elements*) relies on fluent understanding *and* habits of mind, one of which is reasoning ideas through by making sense of them and basing decisions on the best information you have. There are many other useful habits of mind – not least the ability to persevere in the face of difficulties – but that is the subject of a different article.

So, *reasoning is a key component of the problem-solving process* as well as being involved in the development of fluent understanding of the knowledge to be applied.

What are the implications for classroom practice?

- Privilege reasoning over all other activities in the maths classroom.
- Spend more time on discussion and less on recording; it supports conceptual development and formative assessment.
- Independent work that focuses on true problem solving assesses all three aims of the curriculum.
- Ensure all learners believe that their ideas are welcome and useful to the purpose of any discussion, which is for <u>all</u> learners to develop a true and robust understanding of the concepts being learnt.[2]
- Assess learners as mathematicians, asking, "W*hat can they do with the content they have learnt?"*

[2] Some learners may believe that the purpose is 'to be the first to get the correct answer' or 'to avoid getting the wrong answer' but both beliefs discourage participation and sharing of ideas.